Fitness: "The Maximum Likelihood Degree"

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1. Consider the statistical model $M \subset \Delta_5$ parametrized by the rational normal curve:

$$\phi(s,t) = (s, st, st^2, st^3, st^4).$$

   (a) Compute mldeg(M).

   (b) Suppose we observe the data $u = (7, 5, 3, 4, 1)$. What is the MLE?

   (c) How does the MLE change if the data is now $u = (1, 2, 2, 4, 6)$? Can you interpret this in terms of the probabilities?

2. Consider now the scaled version of the previous parametrization:

$$\phi^c(s,t) = (c_1 s, c_2 st, c_3 st^2, c_4 st^3, c_5 st^4).$$

   (a) Generate a random positive scaling $c = (c_1, c_2, c_3, c_4, c_5)$ and compute mldeg(M_c). Does it coincide with your answer in 1.(a)?

   (b) Consider the scaling $c = (1, 7, 17, 17, 6)$. What is now mldeg(M_c)?

   (c) Can you find c such that mldeg(M_c) $= 1$? How about such that mldeg(M_c) $= 2$?

3. Let $1 \leq a \leq b$ be integers. The *Hirzebruch* model $H_{a,b} \subset \Delta_{a+b+2}$ is the image of the parametrization

$$\phi(s,t,x) = (s, sx, sx^2, \ldots, sx^a, t, tx, \ldots, tx^{b-1}, tx^b).$$

   (a) Compute mldeg($H_{a,b}$) for $(a,b) \in \{(1,2),(1,3),(2,2),(2,3),(2,4),(2,5),(3,3),(3,4)\}$.

   (b) Can you guess a closed-form formula for mldeg($H_{a,b}$)?